

Annual Report for NERI Proposal #2000-0109 on Forewarning of Failure in Critical Equipment at Next-Generation Nuclear Power Plants

September 2001

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**ANNUAL REPORT FOR NERI PROPOSAL #2000-0109
ON FOREWARNING OF FAILURE IN CRITICAL
EQUIPMENT AT NEXT-GENERATION
NUCLEAR POWER PLANTS**

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September 2001

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U. S. DEPARTMENT OF ENERGY
under contract DE-AC05-00OR22725

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ABSTRACT

This annual report describes the first year's accomplishments under the NERI2000-109 project. We present a model-independent approach to quantify changes in the nonlinear dynamics underlying time-series data. From time-windowed data sets, we construct discrete distribution functions on the phase space. Condition change between base case and test case distribution functions is assessed by dissimilarity measures via L_1 -distance and χ^2 statistic. The discriminating power of these measures is first tested on noiseless model data, and then applied for detecting dynamical change in power from a motor-pump system. We compare the phase-space dissimilarities with traditional linear and nonlinear measures used in the analysis of chaotic systems. We also assess the potential usefulness of the new measures for robust, accurate, and timely forewarning of equipment failure.

1. INTRODUCTION

This NERI Project began in August 2000. The project has three tasks. The first (current) project year involves only Task 1, namely development of nonlinear prognostication for failures in critical equipment at nuclear power facilities. Examples of such equipment include blowers, compressors, fans, vacuum pumps, cooling units, generators, invertors, motor generators, governors, couplings, gearboxes, motors (electric, hydraulic, pneumatic), pumps, valve operators, and turbines. This annual report describes the work status for the first year of the project, spanning August 2000 through August 2001. Tasks 2-3 span the second (FY 2002) and third (FY 2003) project years, and will not be discussed in this annual report. Section 2 describes the status of the tasks, issues/concerns for each task, cost performance, and status summary of tasks. Section 3 discusses the detailed technical aspects of the work, including the technical background, traditional linear and nonlinear analysis, phase-space dissimilarity analysis, validation of the dissimilarity measures for model data, and analysis of equipment data. Section 4 presents the conclusions of this year's work and summarizes the expectations for the second year's work.

2. PROJECT NARRATIVE

This narrative begins by explaining the project subtasks from the NERI2000-109 proposal for the first project year. These subtasks are listed in the same order as in the proposal, for easy reference.

Task 1.1 of our proposal is as follows. *A database of diagnostic data sets will be assembled from historical or newly acquired data. We will first locate occurrences of the most significant failures, and then assemble the associated diagnostic data. This data will begin with the failure occurrence, and extend backward in time to the baseline period. The data sets will then be analyzed via linear measures for obvious trends.*

Under Task 1.1, time and funding constraints for the first project year did not allow long-term failure monitoring of nuclear power plant equipment. Instead, we acquired new data via accelerated failure tests by seeding specific faults in test equipment. In consultation with the Oak Ridge National Laboratory (ORNL), DE&S constructed a test plan, which includes a summary of important-to-safety equipment in nuclear power plants; the choice of two test modes (unbalance and misalignment); the DE&S testing facility; detailed specifications of the equipment to be tested; the test protocol; and specifications of the data acquisition equipment. Appendix A contains the full test plan, which was completed in the third project quarter. DE&S provided sample test data to ORNL for preliminary analysis. ORNL analyzed this data and found a rich set of nonlinear features. The sampling rate was adequate (12.5 KHz), but the number of data points was too small (16,384 points). ORNL needed voltages and currents from all three phases of the three-phase electric motor, for conversion to instantaneous power. ORNL also needed longer datasets. These requirements necessitated an upgrade to the data acquisition system (Emax by PdMA Corporation), causing some delay while PdMA modified their software. DE&S subsequently received the upgrades and provided the test data for the two test sequences. Total payments to DE&S were \$49,906.40 under the subcontract (versus an allocated cost of \$50K), as follows: \$5,536.00 on December 7, 2000, for preliminary test data; \$6,529.20 on January 4, 2001, for testing options; \$11,139.28 on February 12, 2001, for the test plan; \$14,436.32 on March 16, 2001, for equipment specifications; \$12,265.60 on June 6, 2001, for test data.

Task 1.2 of our proposal is as follow. *The diagnostic data from Task 1.1 will be analyzed for the adequacy of data quality for subsequent nonlinear analysis. ORNL experience indicates that inadequate data quality produces inferior or unusable results. This analysis will evaluate the data-sampling rate,*

digitization precision, number of points per dataset, frequency response of the sensors, and related elements. Adequate quality data typically has $\approx 20,000$ data points, sampled at ≈ 10 times the fundamental rotational frequency over ≈ 1 second at ≈ 6 bits of digitization precision. If the existing historical diagnostic data is determined to be of insufficient quality, changes in the data acquisition methods will be instituted to produce data that is capable of being analyzed by the nonlinear methods. This task will be performed by ORNL.

Under Task 1.2, ORNL performed quality checks of the test data. The table below summarizes the data quality checks that we performed. ORNL identified three misalignment datasets with ranges of instantaneous power that far exceeded the others. DE&S confirmed these findings, and determined that the problem was due to a memory limitation in the Emax system for more than two sequential datasets. DE&S corrected the problem by rebooting the Emax system. DE&S provided replacement datasets of adequate data quality to ORNL.

<u>Brief description of the data quality check for each dataset</u>	<u>For good quality data, result should be</u>
▪ proper number of data points	500,000
▪ any intervals(s) with unchanged signal amplitude	no
▪ adequate sampling rate	no
▪ excessive periodic content	no
▪ excessive noise	no
▪ saturation at high/low limits (indicator of improper amplification)	no
▪ consistent signal amplitude across multiple datasets in the test	yes

Task 1.3 of the proposal is as follows. *Each set of adequate-quality diagnostic data from Task 1.2 will be analyzed with the nonlinear paradigm to determine the presence of a statistically significant condition change indication. This analysis will also determine the characteristics of the PS-DF associated with each specific failure type. This task will be performed by ORNL.*

Task 1.4 of the proposal is as follows. *A library of PS-DF types and their correlated failure types will be developed for subsequent correlation to unknown failures by means of the nonlinear characteristics. This task will be performed by ORNL.*

Task 1.5 of the proposal is as follows. *The extent of PS-DF changes, via the measures of dissimilarity, will be associated with the time remaining until observed failure for the observed failure events. This correlation will be used subsequently to indicate the assessment of remaining condition of the equipment. This task will be performed by ORNL.*

ORNL used a research-class FORTRAN code that performs Tasks 1.3-1.5 as an integrated sequence of algorithmic operations on both the misalignment data and the imbalance data from DE&S. The analysis converts time-serial, process-indicative data into a discretized phase-space (PS) representation. The resulting distribution function (DF) captures the location and occurrence frequency for the nonlinear process dynamics. Dissimilarity measures indicate departure of the test case DF from the baseline DF as an underlying system parameter changes. Forewarning of failure corresponds to a statistically significant rise in dissimilarity, as to the desired outcome of Task 1.3. The library under Task 1.4 is formed by the sequence of PS-DFs for the misalignment and unbalance tests. Correlation of the dissimilarity measures with the failures was performed by reference to the ISO standards 2372 and 3945, as the desired outcome for Task 1.5. Sections 3.2 and 3.4 of this report describe this analysis in detail.

Task 1.6 of the proposal follows. *This task will involve the robust implementation of the nonlinear analysis algorithms for near-real-time analysis of equipment data. Specifically, ORNL will implement the nonlinear paradigm on a desktop computer, which will be placed at an appropriate DE&S site for use by*

the reactor operators there. This mode of on-site data acquisition and diagnosis will be similar to the mode of operation for equipment prognostication at an advanced nuclear reactor. This task will focus on algorithm changes that minimize the memory requirements and maximize computational speed.

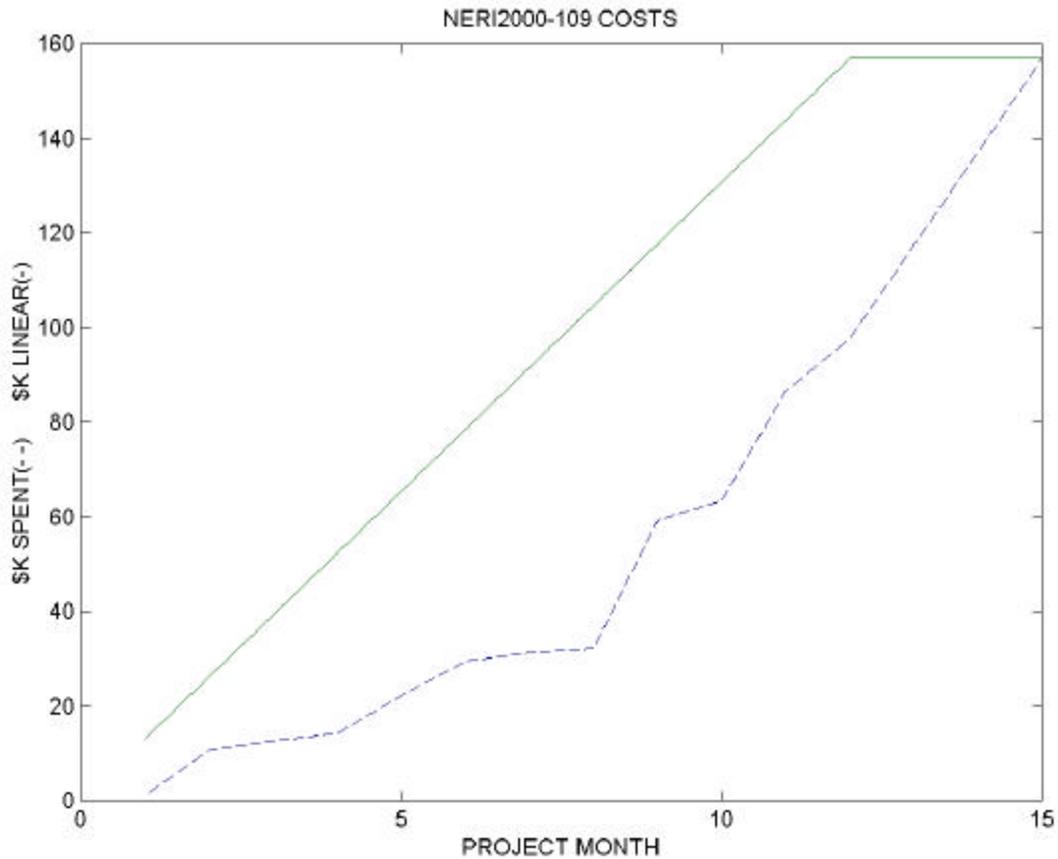
ORNL procured and set up a Win2000 1.3GHz Athlon PC with 1.5GB of memory and two 81 GB hard drives. The PC also has Ethernet capability, keyboard, mouse, video board, read-only CD-ROM, and data archival capability (100MB Zip drive and re-writable CD-ROM). An existing 15" monitor at ORNL was used to display the results. A new Compaq FORTRAN compiler was procured for this computer. Procurement costs totaled \$3,026. Sample analyses for Sec. 3.6 from an existing 500 MHz Pentium II computer were reproduced exactly on the new machine, demonstrating the robustness of the FORTRAN algorithm in moving across processors (Pentium II to Athlon) and different operating systems (WinNT to Win2000). The new PC is 2.42 times faster than the older machine, consistent with the proportionately faster processor speed ($1.3\text{GHz}/500\text{MHz} = 2.6$). The new PC has not been transferred to DE&S because on-site data acquisition of operational data is not anticipated for the second project year. This new computer will significantly enhance ORNL's analysis capability for this project.

2.1 ISSUES AND CONCERNS

Analysis of preliminary test data identified a need for longer datasets and more data channels, as explained in the narrative for Task 1.1. ORNL requested that DE&S obtain an upgrade the Emax software from PdMA Corporation to meet these requirements, causing a delay in the test plan and diagnostic data acquisition, as described above. Project spending has not risen linearly, also due to this delay in data acquisition. Consequently, we anticipate that \$30–40K of FY 2001 funds will not be spent in the first project year. We have requested that Phil Wong (Oakland Operations Office) authorize carry-over of these funds into FY 2002. We expect to use this carry-over funding for more detailed nonlinear analysis of the test data.

2.2 COST PERFORMANCE

We received \$157,000 for the first project year on August 18, 2000. Total costs through the fourth fiscal quarter of the first project year (August 2000 through August 2001) are \$97,725. We anticipate that spending through the end of FY 2001 will leave \$30–40K of carry-over into FY 2002, as shown in the plot of project costs versus time below. The dashed (- -) curve from months 12–15 shows subsequent expected costs, versus nominal linear spending versus time in the solid curve (—).



2.3 STATUS SUMMARY FOR FIRST PROJECT YEAR

Milestone/task description	Planned completion date	Actual completion date
Task 1.1: ORNL set subcontract in place for DE&S	09/00	10/00
DE&S provide preliminary test data to ORNL	09/00	02/01
DE&S construct test plan for accelerated testing	11/00	04/01
DE&S provide datasets to ORNL	01/01	06/01
Task 1.2: ORNL analyze quality of DE&S test data	02/01	06/01
DE&S provide replacement datasets for any found inadequate	02/01	06/01
Task 1.3: ORNL perform condition change analysis on data	08/01	08/01
Task 1.4: ORNL construct library of nonlinear condition change signatures	08/01	08/01
Task 1.5: ORNL correlate condition change to approaching failure	08/01	08/01
Task 1.6: ORNL procure new computer	08/01	05/01
ORNL implement nonlinear analysis software on new PC	08/01	06/01

3. TECHNICAL APPROACH AND RESULTS

3.1 BACKGROUND

The Advanced Technology Program (ATP) of the National Institute of Standards and Technology (NIST) held a workshop on Condition-Based Maintenance (CBM) during its November 17-18, 1998 fall meeting in Atlanta, Georgia [NIST, 1998]. Workshop participants identified three technical barriers to widespread CBM: (i) the inability to predict the remaining useful life of a machine accurately and reliably; (ii) a lack of continuous machine monitoring; and (iii) the need for decision systems to learn impending failures, and to recommend what action to take. These barriers could potentially be addressed through innovations in three technical areas: (i) prognostication capabilities, (ii) cost effective sensor and monitoring systems, and (iii) reasoning or expert systems. Decision models should accommodate changes in mission compliance, operational environment, economic rules, priority assessments, and functional requirements. The need for decision infrastructure is being addressed by a separate NERI project [Harmon, et al., 2001] that includes DE&S, which is also collaborating on this NERI project.

This NERI2000-109 project addresses the first need for technical innovation (prognostication) via nonlinear analysis of equipment operational data. The NIST/ATP workshop [NIST, 1998] identified high quality diagnostics and sensor information as essential for prognostication. Indeed, data is needed not only for prognostication, but also for training and validating the decision methodology. Workshop participants placed a very high priority on quality and completeness of data sets. The barriers to achieving these goals include: (a) incomplete understanding of the evolution of faults and how they effect equipment; (b) non-robust state-based modeling techniques to develop understanding of physics of failures (reduced order modeling); (c) and lack of predictive methodologies for unsteady signatures that are indicative of physics-based failure modes; (d) ignorance about controlling parameters, which hampers development of accurate models; and (e) unavailability of test facilities, especially replication of the real operating environment. Our NERI2000-109 project addresses items (a)-(c) by quantifying the (non-stationary) condition change in test equipment as a sequence of robust nonlinear statistical signatures for progression of a (seeded) fault in specific test equipment. This project addresses item (d) by associating the change in the controlling parameter (seeded fault) with the equipment response. This project addresses (e) by tests of nuclear-grade equipment at the DE&S facilities, which are very similar to real plant conditions. We use hypothesis testing to demonstrate these capabilities, as discussed in detail below.

We expect that regulatory criteria from the U.S. Nuclear Regulatory Commission (NRC) will continue to govern operation and safety in next generation Nuclear Power Plant (NPP). The NRC identifies three strategic areas for the reactor oversight process [NRC, 2000]: reactor safety, radiation safety (including occupational and public safety) and safeguards (physical protection). Reactor safety relies on mitigating systems, barrier integrity, and emergency preparedness to respond to initiating events (unplanned reactor shutdowns, loss of normal reactor cooling after an unplanned shutdown, and unplanned events that result in significant changes in reactor power). Failure prognostication is intended primarily to forecast initiating events in operational equipment, and secondarily to forewarn of failures in mitigating (safety) systems.

Plant processes evolve from normal to abnormal conditions with an accompanying display of rich dynamics, including multiple time scales, quasi-periodicity, nonlinearity, and chaos. Usually, such systems: have many components with hierarchical structure, are driven by various competing forces, and interact strongly with noisy and/or nonstationary environments. Quantitative analysis of the corresponding time serial data has been a difficult and frustrating problem for diagnosis of the degradation, fault, or failure. Key issues include: (i) lack of a proper (physical) model, forcing the analyst

to view signals as generated by a black box whose internal mechanism is either poorly understood, or not understood at all; (ii) non-stationary signals, i.e., with statistical properties that change significantly over the observation period with changes not known *a priori* and not explicitly advertised; (iii) nonlinear structure of various component dynamics and their complex, intricate interconnection, rich in feedbacks and hysteresis; (iv) rarely functioning at steady state, and more typically occurring far from equilibrium via continuous feedback-control loop(s) to adjust to changing conditions.

3.2 APPROACH

One of the most important problems encountered in nonlinear time-series analysis is the appropriate characterization of features and events in nonlinear systems' dynamics. Often these features are either described by several different quantities or do not have a precise definition at all. The former category includes: (content of) information, (relative) entropy, and synchrony. Examples of the latter group are: coherence, patterns, or complexity. These features may have various origins, such as nonstationarity, nonlinearity, nonequilibrium, and intertwining of length- and time-scales. The presence of any one of these factors frequently introduces erratic fluctuations, patchiness, lack of obvious structure, or other irregularities. Previously, these irregularities have been neglected as noise without much structure and meaning. Recent advances in nonlinear science have facilitated the interpretation of intermediate and small-scale details as *bona fide* structure, with significant information about the underlying dynamics. Analysis of this structure enables a deeper understanding of basic dynamical features of system, and results in more efficient assessment, prediction, prevention, control, and repair of their malfunctions.

We address the forewarning problem within a purely pragmatic approach geared at designing, testing, and implementing such measures. We base this approach on a set of nested assumptions that we retain or discard by: (a) the Occam's razor (i.e. start with a simple explanation before resorting to a complicated one); (b) consideration of falsifiable hypotheses only; and (c) acceptance of operationally realizable tests only. In a more or less decreasing order of generality, the assumptions underlying our approach are:

- (i) For a broad range of circumstances, the motor-pump system behaves as a finite-dimensional nonlinear, possibly chaotic dynamical system. This assumption underlies all efforts of modeling such systems by a system of coupled nonlinear evolution equations, for which relevant dynamics occur on a bounded, finite dimensional region of the phase space (PS), called an attractor. Moreover, under assumption (i), we do not attempt to answer questions about nonstationarity or nonequilibrium. Indeed, statistical tests for stationarity produce a binary result, namely, they indicate whether a change occurred, but provide no information about the extent of departure from one state to another. Stationarity tests also have limited value for inherently nonstationary processes that undergo changes in dynamics. For such nonstationary processes, a measure of dissimilarity that quantifies the "distance" between attractors turns out to be more useful [Schreiber, 1997; Moeckel and Murray, 1997; Schreiber, 1999]. This approach is closely related to testing hypotheses of chaotic fluctuations by comparing the "spatial distance" in phase space between observed time series and theoretical attractors [Bjornstad and Grenfell, 2001; Cushing et al. 1998]. Such phase-space comparisons also provide a robust criterion for estimating model parameters [Bjornstad and Grenfell 2001; Ellner and Seifu, in press].
- (ii) Time-serial power data captures the main features of nonlinear equipment dynamics. Recent studies show that different observables do not capture the same amount and/or quality of information [Letellier *et al.*, 1998]. Obviously, this result has momentous implications for forewarning analysis. In the absence of a model, the "correct" choice among apparently equivalent channels can be assessed only *a posteriori*.

- (iii) The first two assumptions are easy to understand and are well documented in the literature, allowing the *use of nonlinear dynamical methods for time-series analysis*. Global aspects of the equipment dynamics can be adeptly captured, characterized, and discriminated by nonlinear descriptors such as Lyapunov exponents, Kolmogorov entropy, correlation dimension, etc. [Qu et al., 1993]. Straightforward methods exist [Eckmann and Ruelle, 1985; Abarbanel, 1996; Cover et al., 1997] for discriminating between regular and chaotic motion, or for detecting the transition between these regimes. However, distinguishing different chaotic regimes can be very difficult, especially when data are limited and noisy.
- (iv) The PS parameters can be adequately chosen for equipment failure forewarning. In addition to implicitly relying on the validity of (i)-(iii), this assumption constrains the length and quality of the data.
- (v) No significant correlation exists between the base case and the failure event, and thus no time relationship between the physical state of the base case and event. This assumption simply implies that the characteristic time of the underlying equipment dynamics is much shorter than the time interval between the “normal” (base case) regime and the onset of the abnormal behavior. We shall see in Sec. 3.6 the effect of violating this assumption on model data.
- (vi) A fixed threshold value for all the data sets is sufficient for robust and reliable forewarning. On the one hand, the threshold is easy to understand and modify operationally, but is very difficult to justify by general principles since the very notion of threshold is “in the eye of the beholder”. On the other hand, the results of the analysis depend heavily on the threshold value. Continuing test input and adjustment is necessary for successful practical implementation.
- (vii) Forewarning of an event is indicated by several successive occurrences above threshold within the forewarning window. The same caveats apply to this assumption as in (vi). Here we choose the number of crossings by striking a balance between timeliness and accuracy of forewarning. Within the scope of this study, this judiciousness of this balance is evaluated *a posteriori*.

We systematically tested the validity of the assumptions (iv) and (vii), including various checks in the algorithm development. In particular, we tested these hypotheses one by one, starting with the simplest ones via appropriate analysis of the data, while keeping the others unchanged. If an assumption was found to be false, it was rejected and replaced by a more valid assumption. A conclusive test of assumption (iv) requires statistically significant amounts of standard length data of verified quality for *all types* of equipment failures. Such a test is beyond the scope of the present project. The results of such an analysis would allow a test of more “universal” values for the parameters under assumptions (v) through (vii).

This section is organized as follows. Sections 3.3-3.4 discuss typical traditional linear and nonlinear measures for time series analysis. Section 3.5 explains the phase-space analysis, and Sec. 3.6 presents results of our analysis on model data. Section 3.7 describes the analysis of machine data.

3.3 LINEAR MEASURES

Analysis of time serial data begins with the collection of a process-indicative scalar signal, x , from a dynamical system whose dimensionality, structure, parameters, and regime are usually unknown. This signal is sampled at equal time intervals, τ , starting at the initial time, t_0 , and yields a sequence of N points, $x_i = x(t_0 + i\tau)$. Several linear measures are useful for characterizing the gross features of this data. The first is the mean, \bar{x} , or average over the N data points:

$$\underline{x} = \sum_{i=1}^N x_i \quad (3.1)$$

The second is the sample standard deviation (σ), which follows from Eq. (3.1):

$$\sigma^2 = \sum_{i=1}^N (x_i - \underline{x})^2 / (N-1). \quad (3.2)$$

Equation (3.2) is the second moment about the mean, implying that higher moments are available. Thus, a third linear measure is the third moment about the mean, called skewness, s :

$$s = \sum_{i=1}^N (x_i - \underline{x})^3 / N\sigma^3. \quad (3.3)$$

A fourth linear measure is the fourth moment about the mean, called kurtosis, k :

$$k = \sum_{i=1}^N (x_i - \underline{x})^4 / N\sigma^4 - 3. \quad (3.4)$$

Typical process data have significant values for skewness and kurtosis, but Gaussian random processes have values that are not significantly different from zero [Abramowitz and Stegun, 1965]. A large positive (negative) value of skewness corresponds to a longer, fatter tail of the data distribution about the mean to the right (left). Kurtosis measures the amount of flattening (negative k) or excess peakedness (positive k) about the mean. Another measure applies to both linear and nonlinear systems, and involves counting the number of times (n_c) that the signal crosses the mean value. More specifically, one-half of a wave period is delimited by two successive mean crossings. For $n_c \gg 1$, the average number of time steps per wave cycle (m) as:

$$m = N / [n_c - 1/2] = 2N / (n_c - 1) \approx 2N / n_c. \quad (3.5)$$

This last measure indicates the average periodicity in the signal, or the inverse of the average frequency. Analysis of typical data (below) shows that these measures provide little, if any, discrimination for detection of condition change. We include these measures for the sake of completeness and to show that linear measures are inadequate for prognostication.

3.4 TRADITIONAL NONLINEAR MEASURES

Nonlinear analysis uses the same sequence of time serial data (x_i) to reconstruct the process dynamics. In particular, phase-space (PS) reconstruction [Eckmann and Ruelle, 1985] uses d -dimensional time-delay vectors, $y(i) = [x_i, x_{i+\lambda}, \dots, x_{i+(d-1)\lambda}]$, for a system with d active variables and time lag, λ . The choice of lag and embedding dimension, d , determines how well the PS reconstruction unfolds the underlying dynamics from a finite amount of noisy data. Takens found that, for a d -dimensional system, $2d + 1$ dimensions generally results in a smooth, nonintersecting reconstruction [Takens, 1981]. Sauer *et al.* (1991) showed that, using *ideal data* (i.e. no noise and infinite precision), the first integer greater than the correlation dimension is often sufficient to reconstruct the system dynamics; this result has been confirmed by computing the embedding dimension via the false nearest-neighbors method [Abarbanel and Kennel, 1993; Abarbanel *et al.*, 1993; Cao 1997]. However, too high an embedding dimension could result in overfitting for *real data* with finite length and noise. We further note that different observables

of a system contain unequal amounts of dynamical information [Letellier *et al.* 1998], implying that PS reconstruction could be easier from one variable, but more difficult or even next to impossible from another. As indicated in the discussion of assumptions (i)-(vii), our analysis seeks to balance these caveats within the constraints imposed by the finite length noisy data.

Various nonlinear measures have been defined to characterize process dynamics using the PS reconstruction. [Kantz and Schreiber, 1997; Rezek and Roberts, 1998]. We choose three of these nonlinear measures, against which we compare the dissimilarity indicators. In particular, we use: the first minimum in the mutual information function as a measure of decorrelation time, the correlation dimension as a measure of dynamic complexity, and the Kolmogorov entropy as a measure of predictability. For the reader's convenience, we briefly describe these three measures next.

The mutual information function (MIF) is a nonlinear version of the (linear) autocorrelation and cross-correlation functions and was originally developed by Shannon and Weaver (1949) with subsequent application to time series analysis by Fraser and Swinney (1986). The MIF measures the average information (in bits) that can be inferred from one measurement about a second measurement and is a function of the time delay between the measurements. Univariate MIF measures predictability within the same data stream at different times. Bivariate MIF measures predictability of one data channel, based on measurements in a second signal at different times. For the present analysis, we use the first minimum in the univariate MIF, M_1 , to indicate the average time lag that makes x_i independent of x_j . The MIF, $I(q,r)$, and system entropy, H , are defined by

$$I(q,r) = I(r,q) = H(q) + H(r) - H(r,q) , \quad (3.6)$$

$$H(q) = -\sum_i P(q_i) \log[P(q_i)] , \quad (3.7)$$

$$H(q,r) = -\sum_{i,j} P(q_i, r_j) \log[P(q_i, r_j)] . \quad (3.8)$$

For a window of N points, we denote the Q set of data measurements by q_1, q_2, \dots, q_N , with associated occurrence probabilities $P(q_1), P(q_2), \dots, P(q_N)$. R denotes a second set of data measurements, r_1, r_2, \dots, r_N , with a time delay relative to the q_i values, having associated occurrence probabilities $P(r_1), P(r_2), \dots, P(r_N)$. The function $P(q_i, r_j)$ denotes the joint probability of both states occurring simultaneously. H and I are expressed in units of bits if the logarithm is taken in base two.

The maximum-likelihood correlation dimension, D , is [Takens 1984; Schouten *et al.* 1994a]:

$$D = \left\{ (-1/M) \sum_{ij} \ln[(\mathbf{d}_{ij} / \mathbf{d}_0 - \mathbf{d}_n / \mathbf{d}_0) / (1 - \mathbf{d}_n / \mathbf{d}_0)] \right\}^{-1} , \quad (3.9)$$

where M is the number of randomly sampled point pairs; \mathbf{d}_{ij} is the maximum-norm distance between the (randomly chosen) $i - j$ point pairs, as defined in Eq. (3.11) below. The distance (scale length) \mathbf{d}_n is associated with noise as measured from the time serial data. Note that the distances are normalized with respect to a nominal scale length \mathbf{d}_0 , which is chosen as a balance between sensitivity to local dynamics (typically at $\mathbf{d}_0 \leq 5a$) and avoidance of excessive noise (typically at $\mathbf{d}_0 \gg a$). Here, the symbol a denotes the absolute average deviation as a robust indicator of variability [Schouten *et al.* 1994a] in the data,

$$a = (1/w) \sum_{i=1}^w |x_i - \underline{x}|, \quad (3.10)$$

where \underline{x} is the mean of x_i over the window of N points. The distances \mathbf{d}_{ij} are defined by

$$\mathbf{d}_{ij} = \max_{0 \leq k \leq m-1} |x_{i+k} - x_{j+k}|, \quad (3.11)$$

where m is the average number of points per cycle, as determined by Eq. (3.5).

The Kolmogorov entropy, K , measures the rate of information loss per unit time, or (equivalently) the degree of predictability. A positive, finite entropy is generally considered a clear demonstration that the time series and its underlying dynamics are chaotic. A very large entropy indicates a stochastic (nondeterministic) and therefore totally unpredictable phenomenon. The K-entropy is estimated from the average divergence time for pairs of initially close orbits. More precisely, the entropy is obtained from the average time for two points on an attractor to go from an initial separation $\mathbf{d} \leq \mathbf{d}_0$ to a separation of more than a specific distance ($\mathbf{d} > \mathbf{d}_0$). The maximum-likelihood K-entropy is calculated from the method by Schouten *et al.* (1994),

$$K = -f_s \log(1 - 1/\underline{b}), \quad (3.12)$$

$$\underline{b} = (1/M) \sum_{i=1}^M b_i, \quad (3.13)$$

with b_i as the number of timesteps for two points, initially within $\mathbf{d} \leq \mathbf{d}_0$, to diverge to $\mathbf{d} > \mathbf{d}_0$. The symbol f_s denotes the data-sampling rate.

There are several problems associated with the use of these measures for detection of dynamical change. The most serious is that these nonlinear measures are expressed as a sum or integral over (a region of) the PS, which averages out all dynamical details into a single number. Two (very) different dynamical regimes may lead to very close, or even equal measures. The situation is even murkier for noisy dynamics, in which case reliable determination of the nonlinear measures is next to impossible. The second difficulty arises from the definitions of K-entropy and correlation dimension in the limit of zero scale length. However, all real data have noise and even noiseless model data is limited by the finite precision of computer arithmetic. Thus, we choose a finite scale length that is somewhat larger than the noise ($\mathbf{d}_0 = 2a$), at which to report the values of K and D , corresponding to finite-scale dynamic structure. Consequently, the calculated values of K and D have smaller values than expected for the zero-scale-length limit ($\mathbf{d}_0 \rightarrow 0$) and cannot capture dynamical complexity at length scales smaller than \mathbf{d}_0 . A third difficulty arises from the definition of these nonlinear measures as functionals of the distribution functions. Some of these functionals do not satisfy all the mathematical properties of a distance. In particular, for some of them, symmetry and the triangle inequality may be violated [Quin Quiroga *et al.*, 2000]. Therefore, these measures cannot define a metric in the mathematical sense. They may indicate change, although only in a sense that has to be made precise for each situation.

In an attempt to improve the discrimination power, Thomasson *et al.* (2001) has recently proposed the “recurrence quantification” approach that does not require assumptions about stationarity, length, or noise. Their new measure quantifies the recurrence of sets of points of various lengths that “almost repeat themselves” during the dynamics. It can be viewed somewhat as a generalization of the Poincaré section concept and is designed to detect and characterize “real phenomena” present in the time serial data. Since